3-3 Rules for Differentiation

Learning Objectives:

I can use the Power Rule to find derivatives.

I can use the product and quotient rule to find derivatives.

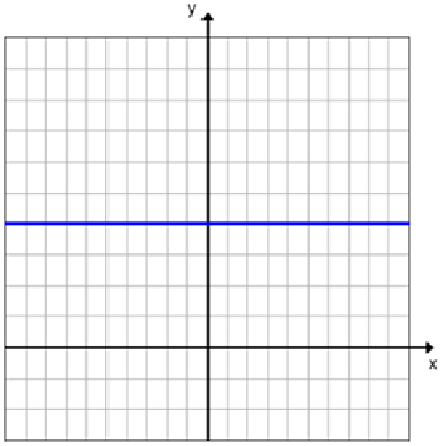
I can find second and higher order derivatives.

Sep 21-12:38 PM

Derivatives of a Constant Function

$$f(x) = c$$

$$f'(x) = 0$$

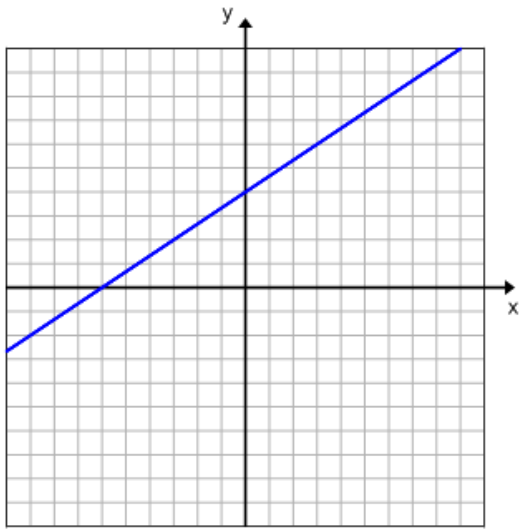


Sep 21-12:38 PM

Derivatives of a Linear Function

$f(x) = mx + b$

$f'(x) = m$



Sep 21-12:38 PM

Power Rule

$$f(x) = x^n$$

$$f'(x) = n \cdot x^{n-1}$$

Sep 21-12:38 PM

Ex1. Find the derivative of the following functions

1.) $y = x^8$

$y' = 8x^7$

2.) $y = 4x^5$

$y' = 20x^4$

3.) $g(x) = 4x^3 + 6x^2 - 5x + 8$

$g' = 12x^2 + 12x - 5$

4.) $f(x) = \sqrt{x}$

$x^{1/2}$

$f = x^{-1/2}$

$f' = \frac{1}{2}x$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$

$f' = \frac{1}{2\sqrt{x}}$

5.) $f(x) = \frac{3}{x^2}$

$f = 3x^{-2}$

$f' = -6x^{-3}$

$f' = -\frac{6}{x^3}$

Sep 21-12:38 PM

Ex2. Does the function have any horizontal tangent lines? If so, where are they?

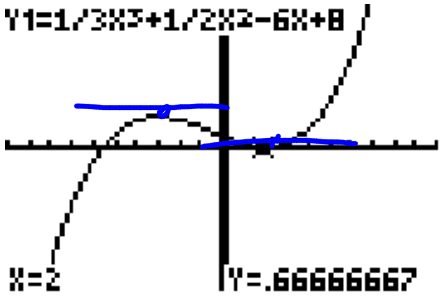
$$g(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-6x+8$$

$g' = x^2 + x - 6$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$x = -3 \text{ & } 2$



Sep 21-12:38 PM

The Product Rule

$$h(x) = f(x) \cdot g(x)$$

$$h'(x) = f' \cdot g + f \cdot g'$$

Sep 21-12:38 PM

Ex3. Find the derivative of the following functions

1.) $f(x) = (2x+3)(x^2+5x-7)$

f g

$y' = f' \cdot g + f \cdot g'$

$f = 2x+3$ $\quad g = x^2+5x-7$

$f' = 2$ $\quad g' = 2x+5$

$y' = 2 \cdot (x^2+5x-7) + (2x+3)\cdot(2x+5)$

$= 2x^2+10x-14+4x^2+10x+6x+15$

$= 6x^2+26x+1$

Sep 21-12:38 PM

2.) $y = (3x+8)\left(\frac{2}{x} - \sqrt{x} - 5\right)$

f

g

$f = 3x + 8$

$f' = 3$

$g = 2x^{-1} - x^{1/2} - 5$

$g' = -2x^{-2} - \frac{1}{2}x^{-1/2}$

$= -\frac{2}{x^2} - \frac{1}{2\sqrt{x}}$

$= -\frac{2}{x^2} - \frac{\sqrt{x}}{2x}$

$y' = 3\left(\frac{2}{x} - \sqrt{x} - 5\right) + (3x+8)\left(\frac{-2}{x^2} - \frac{\sqrt{x}}{2x}\right)$

Sep 21-12:38 PM

The Quotient Rule

$$h(x) = \frac{f(x)}{g(x)}$$

$$h'(x) = \frac{f' \cdot g - f \cdot g'}{g^2}$$

Sep 21-12:38 PM

Ex4. Find the derivative of the following functions

$y' = \frac{f'g - fg'}{g^2}$

1.) $y = \frac{3x^2-5}{2x+1}$ f

$f = 3x^2 - 5$ $\quad g = 2x+1$

$f' = 6x$ $\quad g' = 2$

$$y' = \frac{6x(2x+1) - (3x^2-5)\cdot 2}{(2x+1)^2}$$

$$y' = \frac{12x^2 + 6x - 6x^2 + 10}{(2x+1)^2}$$

$$y' = \frac{6x^2 + 6x + 10}{(2x+1)^2}$$

Sep 21-12:38 PM

2.) $y = \dfrac{x^2-1}{x^2+1}$ (f over g)

$f = x^2 - 1$, $f' = 2x$

$g = x^2 + 1$, $g' = 2x$

$$y' = \frac{f'g - fg'}{g^2}$$

$$y' = \frac{2x(x^2+1) - (x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2}$$

$$y' = \frac{4x}{(x^2+1)^2}$$

Sep 21-12:38 PM

Second and Higher Order Derivatives

$y = x^6 - 3x^5 - 2x^4 + 2x^3 + x^2 - 8x + 1$

$y' = 6x^5 - 15x^4 - 8x^3 + 6x^2 + 2x - 8$

$y'' =$

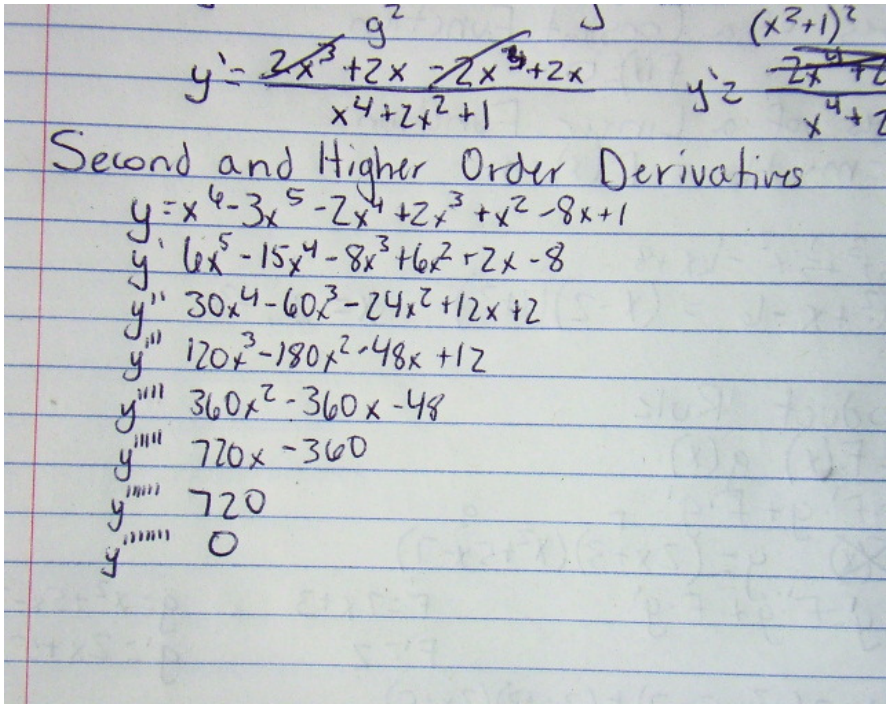
$y''' =$

$y'''' =$

$y''''' =$

$y'''''' =$

$y''''''' =$



Notation for Higher Order Derivatives

First Derivative	Second Derivative	Third Derivative
$f'(x) =$ $y' =$ $\frac{dy}{dx}$	$f''(x) =$ $y'' =$ $\frac{d^2y}{dx^2}$	$f'''(x) =$ $y''' =$ $\frac{d^3y}{dx^3}$
The rate at which the function is changing	The rate at which the rate of change of the function is changing	The rate at which the rate of change of the rate of change of the function is changing

Sep 21-12:38 PM

Homework

pg 124 #5, 6, 7, 9, 11, 15-21 odd, 23, 25, 27, 33, 35, 37-40,46,52

Sep 21-12:38 PM